

Unit 4 – Equilibrium of Forces and Moments (Instructor Narrative)

Engineering Statics in Physics Project
Carnegie Mellon University and Pittsburgh Area Schools

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4.1 A board rests on scales, and people stand on the board – what are the scale readings?

A board and two bathroom scales will be used to show students how to generate a full description of all forces exerted on a body in static equilibrium. That is, students will account for the tendency for forces to cause both linear motion and rotational motion, as captured by the moment – both tendencies will sum to zero for a body in equilibrium. Three configurations will be tried: (i) a board resting on two scales, (ii) a student standing on a board that rests on two scales, and (iii) two students standing on a board that rests on two scales.

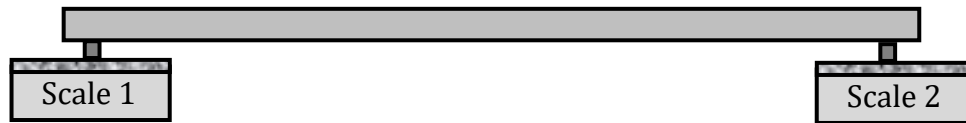
LEARNING OBJECTIVE: Students will be able to identify all of the forces on a body and draw a labeled free-body diagram (FBD). They will be able to use this FBD to account fully for the influences of all forces and moments on the body.

Class Demonstration and Discussion

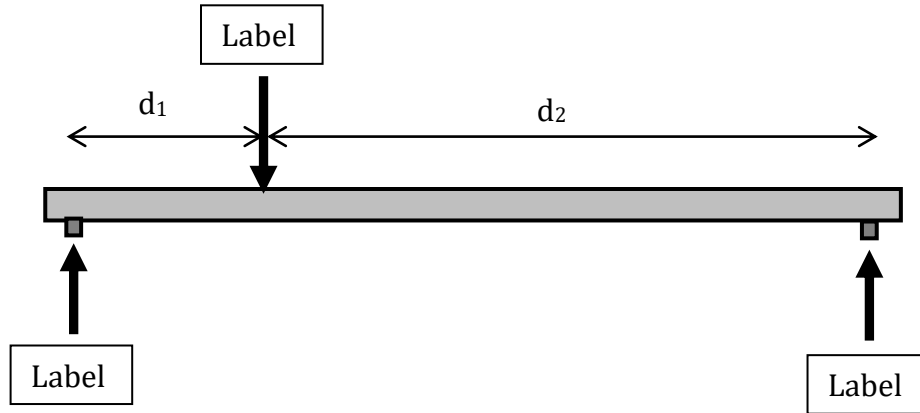
(i) a board resting on two scales: Each time you use a scale, zero it. Weigh the board on each scale separately, and record the mean value for this weight.

Place a strip in the middle of each scale, and rest the board so it extends 2" past center of each strip.

Record the scale readings.



When you draw an FBD that will be used to compute moments and forces, the diagram should include forces and distances. In the sample diagram below, each of the three forces should be labeled, and the two distances should be labeled. Instead of d_1 and d_2 , you would write the actual distances.



Draw an FBD of the board, including board weight, and forces from the strips.



Where do you think the weight of the board acts and why?

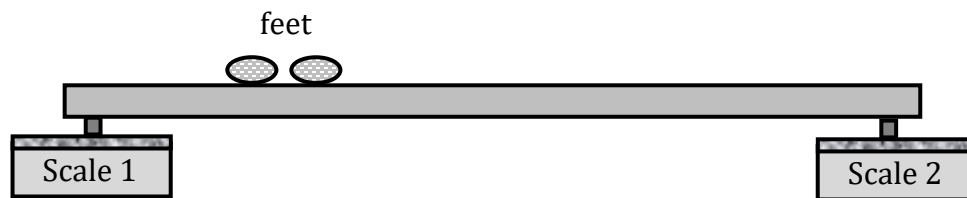
Why is that consistent with equilibrium of the board and the scale readings?

(ii) a student standing on a board that rests on two scales: Weigh the student on each scale and record the mean value of his/her weight.

Place a strip in the middle of each scale, and rest the board so it extends 2" past center of each strip.

Have a student stand with his/her feet comfortably together on the board about $\frac{1}{4}$ to $\frac{1}{3}$ of the distance between the two scales, trying to keep his/her weight evenly distributed between the two feet. Measure the distance from center of each strip to the midpoint between the feet.

Record the scale readings.



Draw an FBD of the board, including the board and student weights, and the forces from the scales. Assume that the weight of board acts in middle of board and student weight acts between the feet.



Compute the sum of forces in the vertical direction, assuming “up” as the positive direction and “down” as the negative direction.

Compute the sum of moments about the left scale strip. Let CCW be the positive direction and CW be the negative direction.

Keeping the same CCW and CW conventions, as above, compute the sum of moments about the right scale strip.

Notice that there is no natural pivot point for this problem. The summation of moments about all points should be zero if the body is in equilibrium. You do not need a pivot point, but you need to take moments about some point. For all bodies in equilibrium, the summation of moments should be zero about any point. But, one can draw conclusions more easily in some problems, when the moments are taken about a convenient point; some of the forces will give zero moment, simplifying the moment summation equation.

Is the resulting sum of moments a nonzero value? If it is nonzero, decide if you have a reasonable value. Is the deviation from zero due to any assumptions you made, measurement errors, or due to an error in your calculation?

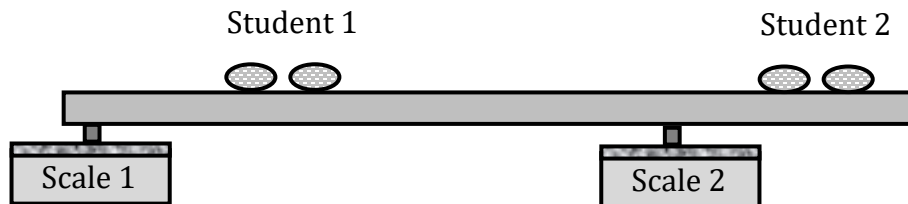
(iii) two students standing on a board that rests on two scales: Weigh each participating student on each scale and record the mean value for each student's weight (or use the weights from earlier experiments).

Place a strip in the middle of each scale, and rest the board so it extends 2" past the center of the left strip, and with approximately $\frac{1}{4}$ of the board length past the right strip. Record the length of the board past the right strip.

Have a student (the heavier) stand with his/her feet comfortably together on the board about approximately 12 inches from the left scale, trying to keep his/her weight evenly distributed between the two feet. Measure the distances between the student's position and the left and right strips. Have a second student (the lighter) stand to the right of the right scale, being careful not to tip the scale. Measure the distance from the right strip to the midpoint between the second student's feet.

Record the scale readings.

What happens to each scale reading if student 2 shifts a little to the left and then a little to the right?



Draw an FBD of the board, including the board and students' weights. Label the forces from the strips as N_1 and N_2 .



Compute the sum of forces in the vertical direction, assuming “up” as the positive direction and “down” as the negative direction. For now, substitute the two unknowns (N_1 and N_2) for the scale readings.

Compute the sum of moments about the left scale strip. Let CCW be the positive direction and CW be the negative direction. Since we are not yet using the scale readings in these calculations, the equation will have one unknown (N_2), because the perpendicular distance for N_1 is 0.

Derive predictions for the values of N_1 and N_2 as follows: solve for N_2 from the summation of moments about the left scale, and then substitute this value for N_2 into the summation of vertical forces and solve for N_1 .

Compare the predicted and measured values of N_1 and N_2 .

	N_1	N_2
Predicted		
Measured (scale readings)		

Are your predicted values in agreement with your experimentally measured values for the scale readings? If they differ, can the difference be attributed to assumptions that you made, measurement errors, or calculation mistakes?

If the students remain in their current positions, how light can the first student (between the scales) be without the board tipping over?

4.2 Two fingers support a long object at one end – what are the forces on the fingers?

A meter stick supported by two metal rods will be used to illustrate relationships between individual moments acting on a body held in static equilibrium.

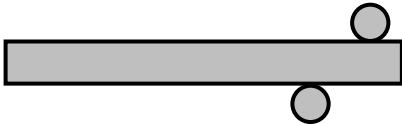
LEARNING OBJECTIVE: Students will be able to identify and qualitatively describe the magnitude and direction of the individual moments on a body held in static equilibrium.

Individual Student Lab Stations

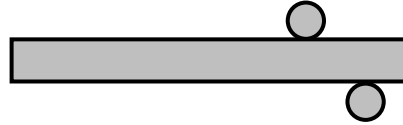
Try to balance a wooden meter stick (as shown below) with two fingers or two metal rods. You might need a second person to make sure that the bar does not slip.

Which arrangement is more successful in supporting the meter stick?

Case A:



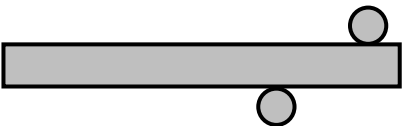
Case B:



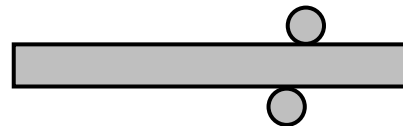
Draw an FBD for each arrangement and use your drawings to rationalize why one arrangement is easier to maintain. [*Hint: consider how the sums of forces and the sums of moments would compare for the two arrangements. By taking moments about either of the rods, the moment summation cannot be zero in one case; taking moments about the stick center does not allow one to draw that conclusion.*]

Balance the meter stick in this way and keeping the lower rod in position, move the upper rod to the left (so that it is nearly above the lower rod as in Case C, below).

Case A:



Case C:



Qualitatively, how do the forces that you apply change as the upper rod is moved to the left?

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Draw a FBD of the meter stick. Use a comparison of the sum of forces and the sum of moments to explain the changes in the forces that you feel as the upper rod is moved. [*Hint: It is easiest to explain the changes if you sum moments about the position of the lower rod.*]

Lastly, imagine keeping the rods a fixed distance apart, and move them together towards the center. Use your FBD and the sums of forces and moments to predict how the forces would change.

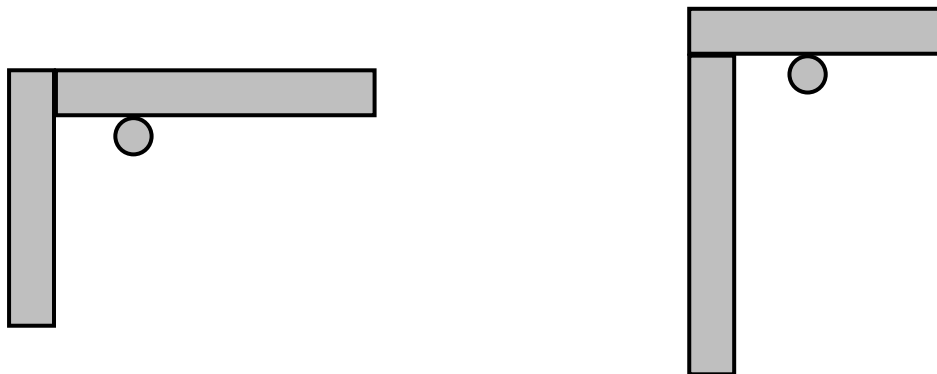
4.3 Finding the location of the center of gravity

An L-shaped bar will be used to demonstrate the concept of center of gravity (CG). The center of gravity is the position at which the total weight acting as a single force would give rise to the same moment for a body as does the body's distribution of weight. This position does not necessarily lie within the body itself. Students will use three methods to determine the center of gravity of the bar: (i) balancing the bar in two orientations on a single rod, (ii) hanging the bar in two orientations from spring scales, and (iii) using the central position and the mass of each rectangle composing the bar to generate a predicted center of mass position for the L-shaped bar.

LEARNING OBJECTIVE: Students will be able to find the center of gravity of simple bodies.

Individual Student Lab Stations

(i) balancing the bar in two orientations on a single rod: Balance an L-shaped wooden bar on a single rod positioned under one side and then positioned under the other side. To locate the two-dimensional location of the CG, record the position of the rod in each case from the left (corner) end. The position readings become the coordinates (d_1 , d_2) of the CG with respect to an axis system centered on the corner of the wooden L-shaped bar.



(ii) hanging the bar in two orientations from spring scales: Hang one side of the L-shaped wooden bar from two spring scales attached to hooks and record the reading of each scale. Measure the length of the side and the position of each scale relative to the left (corner) end. Repeat this process, attaching the hooks to the other side of the “L”. Use each set of scale readings and length measurements to set up a sum of moments that balance to zero. In this way, one can solve for the two position coordinates that describe the CG (d_1 and d_2) relative to the corner of the “L”.

Sum of Forces:

$$\text{scale}_1 + \text{scale}_2 = W$$

Compute the sum moments about the left edge, and set equal to zero:

$$\text{scale}_1 \cdot \text{dist}_1 - W \cdot d_1 + \text{scale}_2 \cdot \text{dist}_2 = 0$$

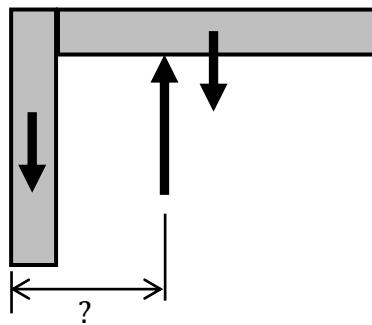
With the bar rotated 90° ,

$$\text{scale}_1 \cdot \text{dist}_1 - W \cdot d_2 + \text{scale}_2 \cdot \text{dist}_2 = 0$$

(iii) using the central position and the mass of each rectangle composing the bar to generate a predicted center of mass position for the L-shaped bar:

Measure and record the area of each component rectangle that makes up the L-shaped bar (area_1 and area_2). If one assumes that the mass is uniformly distributed for each rectangle, and the metal parts are neglected, then the CG of each rectangle component is located at its center. Place the bar with one side oriented horizontally and the other vertically as shown below. Record the distance of each rectangle's center from the left (corner) end of the bar (dist_1 and dist_2). For area 1, the weight equals $W_1 = (\text{area}_1)(\rho g)$ (thickness), where ρ is the mass density. In terms of the distances and the areas write down the summation of moments about the left (corner) edge and set it equal to zero. All terms contain (ρg) (thickness), which can be cancelled. Solve for the distance d_1 of the center of mass from the left (corner) edge. Evaluate in terms of the measured areas. Repeat for the other side of the L.

$$d_1 = [\text{area}_1 \cdot \text{dist}_1 + \text{area}_2 \cdot \text{dist}_2] / [\text{area}_1 + \text{area}_2]$$



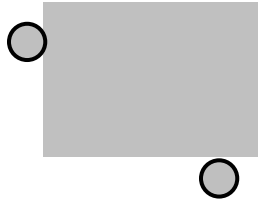
4.4 Combinations of horizontal and vertical forces

A plexiglass rectangle and a wooden L-shaped bar will be used to test and examine the outcome from applying a combination of horizontal and vertical forces. Students will construct FBD's and use the sums of forces and moments to analyze and compare each system. Three cases are tried: (i) holding a rectangular-shaped body with two rods, (ii) holding a rectangular-shaped body with three rods, and (iii) holding an L-shaped body with three rods.

LEARNING OBJECTIVE: Students will be able to draw FBD's that are consistent with situations in which a combination of horizontal and vertical forces are applied to a body. They will be able to use their FBD's to construct sums of forces and moments that can be used to explain whether the body can be held in static equilibrium by these forces.

Individual Student Lab Stations

(i) holding a rectangular-shaped body with two rods: Try to hold the Plexiglass rectangular body with two aluminum rods in a level orientation as shown. Can the rods hold the plate in the orientation as shown? Why or why not?



Draw an FBD for the rectangular body. Label the forces in your diagram as F_{left} , F_{bottom} , and W (for the rectangle's weight).

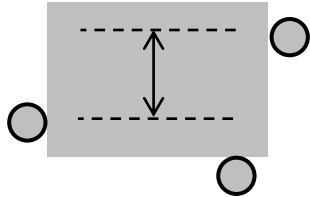


By setting up a (symbolic) sum of forces, one can see that the force F_{left} must be zero. The remaining forces F_{bottom} and W give a net moment if the bottom rod is not below the plate center.

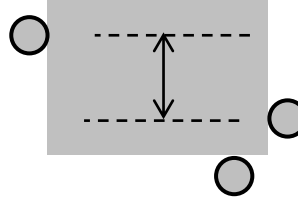
Note: this trial, as well as the trials below, is easily performed by laying the bottom rod on a level surface and resting the rectangular body on this rod before applying the rest of the forces.

(ii) holding a rectangular-shaped body with three rods: Again, keeping the plate level, but now with two of the rods on opposite sides of the rectangle, as shown below, decide which arrangement succeeds in keeping the plate balanced and level?

Case A



Case B



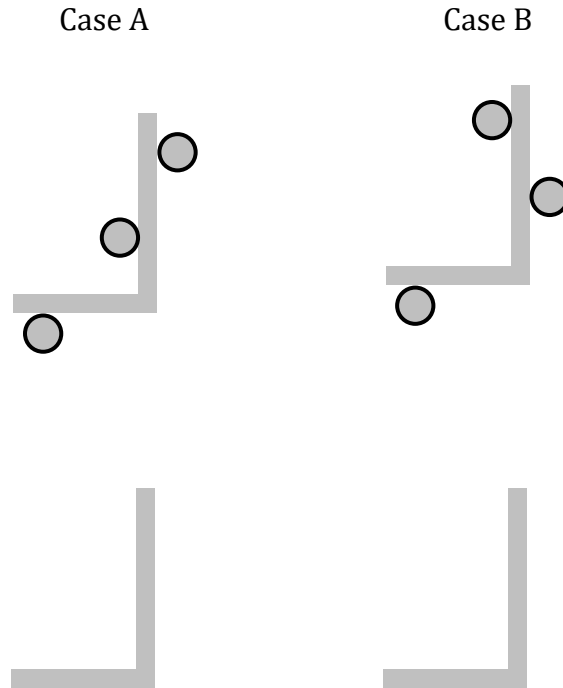
With the bottom rod in one position near the corner, increase and decrease the vertical separation of the forces applied to the sides of the rectangle. How do the magnitudes of the forces of the rods on the sides change as you vary their vertical separation?

Draw an FBD for each case. Label the forces as F_{left} , F_{right} , F_{bottom} , and W (for the rectangle's weight).



Set up sums of forces and a sum of moments (symbolically) for each case that can be used for comparison, using the force and distance labels from the FBD in place of actual measurements. Comparing the FBD's and the sums of moments, explain: 1) which arrangement works and 2) the effect of vertical separation between the side forces. *[Hint: use summation of forces to equate the two horizontal forces and to equate the two vertical forces. Summation of moments about any point, together with the force simplification, will show the arrangement that works and which lengths are important.]*

(iii) holding an L-shaped body with three rods: Hold an L-shaped body with three rods. Place the lower rod at different points along the bottom, and note which arrangement of the other rods succeed in maintaining static equilibrium. Assume that there is no friction. Draw an FBD for each case and use a comparison of the sums of moments to explain what you find.



Note: again take advantage of summation of vertical and horizontal forces to equate forces. Then, if one takes the sum of moments about the position of the bottom rod, it is evident that in Case A one can apply forces on the two upper rods in a way that the CW moment from the weight is balanced by the net CCW moment from the two upper rods, which apply equal forces. In Case B, the moments from the two upper rods have a net CW sense, and are unable to balance the moment of the weight. Finally, the bottom rod can be moved to the right, and eventually Case B maintains equilibrium. This occurs when the bottom rod just passes the center of gravity of the L-shaped body. If there is substantial friction, which is neglected in this analysis, then other arrangements might be able to maintain equilibrium.

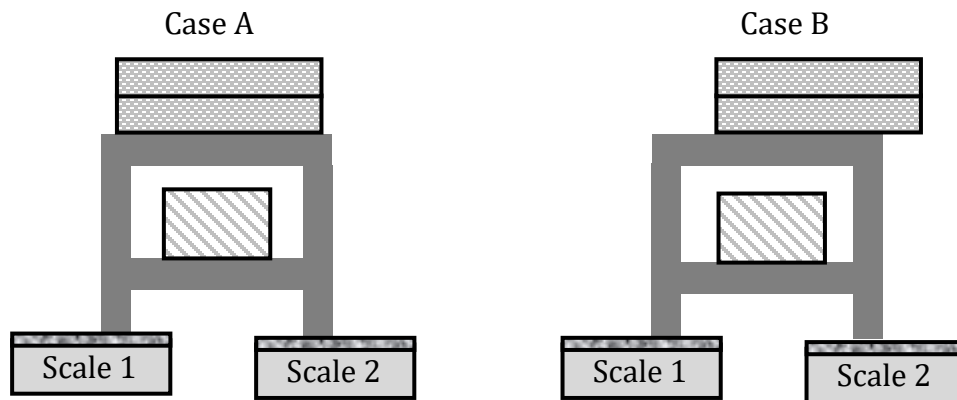
4.5 Horizontal and vertical forces: potential for tipping

A tipping stand, two bathroom scales and some weights will be used to develop a strategy for analyzing the forces involved in tipping an object. Students will practice combining horizontal and vertical forces to predict whether an object will tip or slip in response to a horizontally applied force. Four situations will be examined: (i) shifting weights to one side of a tipping stand that rests on two bathroom scales, (ii) applying a horizontally directed force to a person standing on two bathroom scales, (iii) applying a horizontally directed force to a tipping stand that rests on two bathroom scales, and (iv) altering the vertical position of the horizontally applied force on the tipping scale.

LEARNING OBJECTIVE: Students will be able to predict whether a horizontally applied force will cause an object to tip or to slip.

Individual Student Lab Stations

(i) shifting weights to one side of a tipping stand that rests on two bathroom scales: Place a tipping stand evenly onto two scales as shown below. Put some weights (~10 lb) on the lower shelf of the tipping stand and adjust the weights and the stand's position so that the scale readings are same. Next, put some textbooks on top of the stand. Note any changes in the scale readings when the books are shifted towards one side.

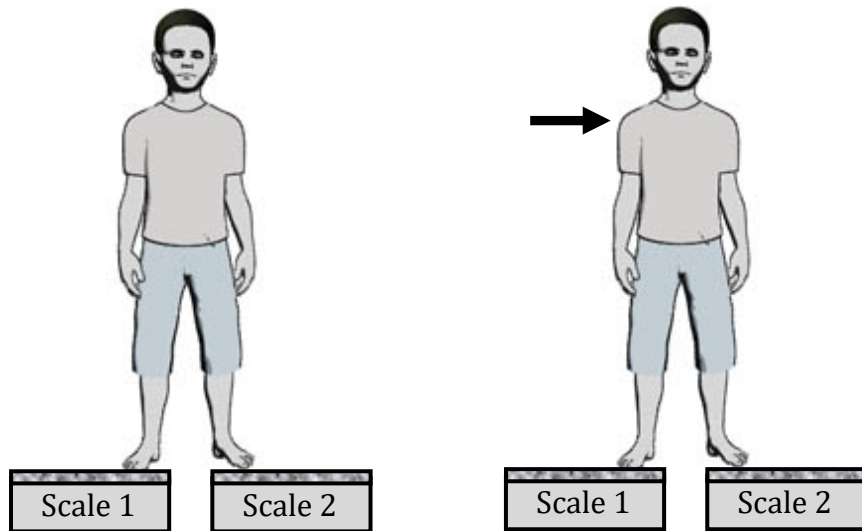


Draw an FBD for the stand in each case.



Set up a sum of forces and a sum of moments (symbolically) for each case that can be used for comparison, using the labels from the FBD in place of actual measurements. Comparing the FBD's and the sums of forces and moments, explain how the scale readings shifted when the books were shifted to one side.

(ii) applying a horizontally directed force to a person standing on two bathroom scales: Have a person stand evenly on two scales so that the scales have the same readings. This person should tense his muscles so that his body does not shift at all. Let another student apply a horizontal force to this person's shoulder and note the change in scale readings.

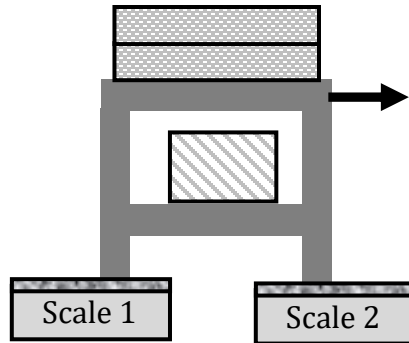


Draw an FBD for the person when he/she is pushed:



Set up a sum of forces and a sum of moments (symbolically), using the labels from the FBD in place of actual measurements. Use your FBD and the sums of forces and moments to explain the observed change in scale readings when the student is pushed.

(iii) applying a horizontally directed force to a tipping stand that rests on two bathroom scales: Position the weights and books so the scales again have the same readings. To prevent slipping, place high friction fabric between the scales and the tipping stand. Use a spring scale to apply a horizontal force as shown in the diagram below.



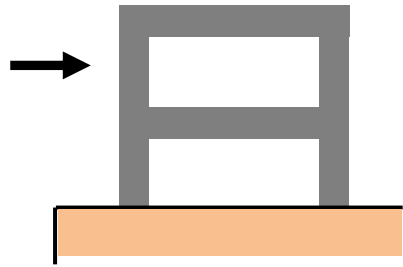
Draw an FBD for the stand.



Set up a sum of forces and a sum of moments (symbolically), using the labels from the FBD in place of actual measurements. Use your FBD and the sums of forces and moments to predict the scale readings for a given value of the horizontally applied force.

Note: You can predict the vertical force of each scale on the stand (i.e. the individual scale readings), but you will not be able to predict the horizontal force of each scale separately.

(iv) altering the vertical position of the horizontally applied force on the tipping scale: Place the tipping stand on a fabric resting on a flat surface. With your hand, apply a horizontal force to the side as shown below. Increase this force until the stand either slips or tips. Alter the vertical position of the horizontally applied force and locate the position at which tipping changes to slipping (or from slipping to tipping).



Compare the “tip-slip transition height” for two types of fabric: a) high friction fabric and b) low friction fabric. Why does the tip-slip transition height change when the fabric type changes? Develop a general method for predicting whether the stand will slip or tip as the horizontally applied force is increased.

Note: For some fabrics or surfaces with low enough friction, the tipping stand is not tall enough to tip no matter where on its side you apply the horizontal force. The general method of predicting the tip-slip transition height requires one to define a friction coefficient between the stand and the surface. Then, summation of forces and moments must be applied. The horizontal forces of the fabric on the two feet of the stand cannot be determined separately, just their sum, but the vertical forces can. Slip occurs if the net friction force reaches μN before the normal force goes to zero.

Force x: $F_{\text{push}} - F_1 - F_2 = 0$

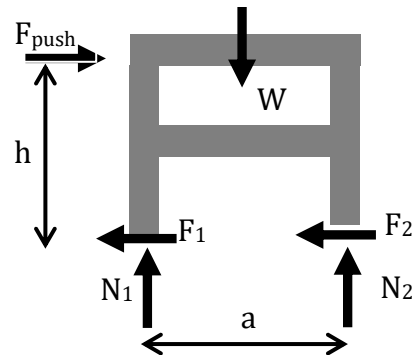
Force y: $-W + N_1 + N_2 = 0$

Moment about right foot of stand:

$$- F_{\text{push}} (h) + W(a/2) - N_1 (a) = 0$$

Slips when $F_1 + F_2 = \mu_s(N_1 + N_2)$

Solve: $F_{\text{push}} = F_1 + F_2$, $N_1 + N_2 = W$



Compare F_{push} to make slip and F_{push} to make $N_1 = 0$ (tip)

4.6 Distributed forces between two bodies in contact and position of single force that represents distribution

Two bodies can exert forces on each other that are distributed over the length (or area) of contact. That distribution is represented by a single force. Comparison of the readings of two bathroom scales will be used to signal that the location of the single normal force between the two bodies in contact can shift to maintain a body in static equilibrium. Two situations will be analyzed: (i) a student who shifts his/her center of gravity and (ii) a tipping stand subjected to a horizontally applied force. In both cases the normal force, between the feet and the base or between the tipping stand and the base, shifts.

LEARNING OBJECTIVE: Students will be able to use the readings from two bathroom scales to arrive at a detailed description (magnitude, direction and position) of the normal force acting on a body at rest.

(i) a student shifts his/her center of gravity: Place two-by-four tracks across two scales and have a student stand on the tracks. Let the student shift his/her weight so that (1) the scales show equal readings (2) the front scale reading is much greater than the rear scale reading and (3) the rear scale reading is greater than that of the front scale.

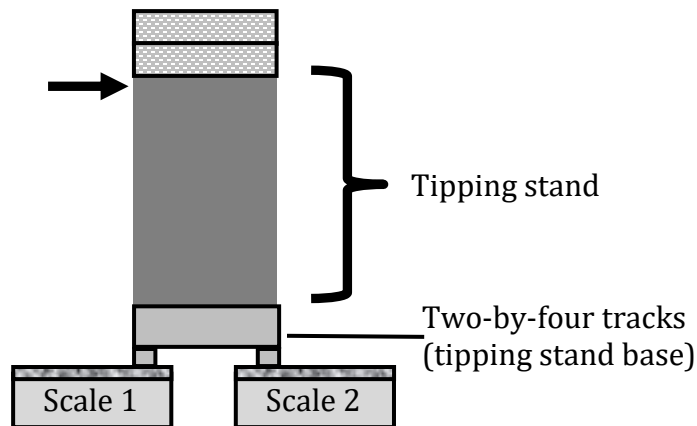
What is changing?



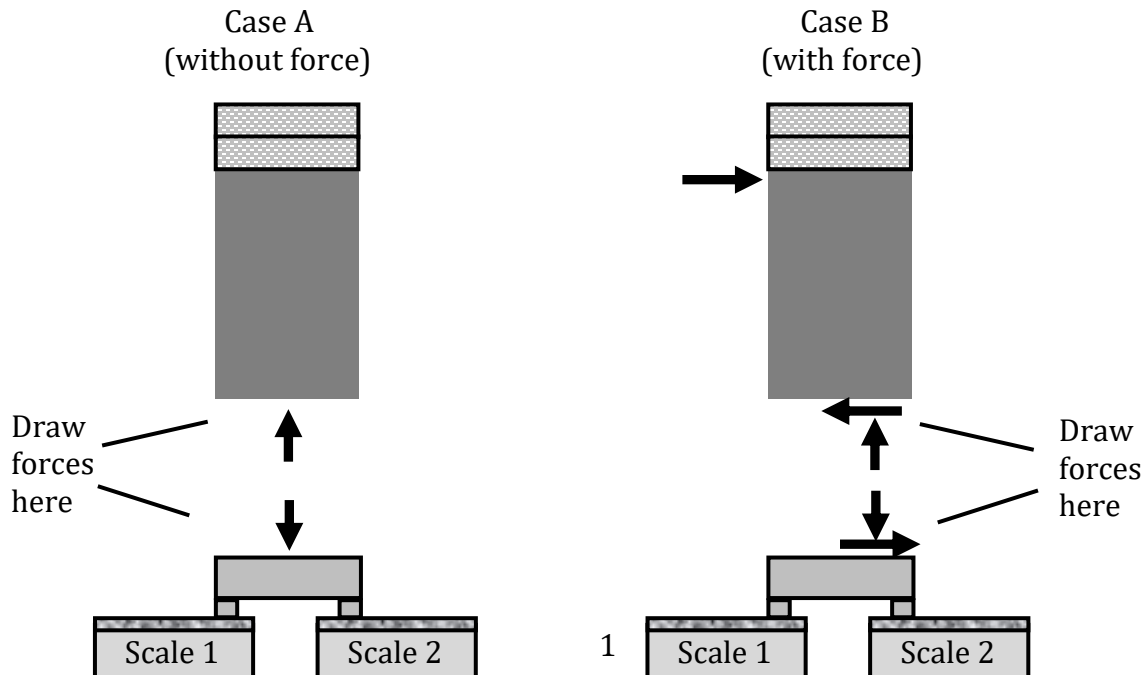
As the student shifts, how is the force between his/her feet and the tracks changing (consider its magnitude and its position)?

In any single position of the body, given the two scale readings, how could you determine the magnitude of the force between the feet and the tracks and the position of this force?

(ii) a tipping stand subjected to a horizontally applied force which causes the stand's weight to shift: Place the tipping stand sideways on the two-by-four tracks (now serving as the tipping stand base). Place weights in the stand, so that the scale readings are equal. Apply a horizontal force to the tipping stand, and notice any changes in the scale readings.



Compare two cases: a) with a horizontally applied force and b) without the horizontally applied force. Draw the FBDs of the tipping stand and the tipping stand base. Be specific about how you draw the forces between the stand and base (*see diagram*).



Justify why the forces you drew in the two cases differ. Explain why the different positions of the normal force make sense in terms of the sums of forces and moments for the tipping stand.

Explain why the different positions of the normal force are consistent with the changes in the scale readings when the horizontal force was applied.

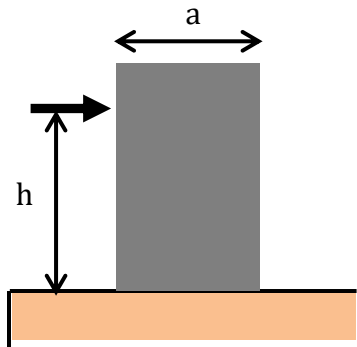
What would eventually happen as the horizontally applied force is increased?

4.7 Potential for slip or tip with distributed forces

When a horizontal force is applied to a box resting on a surface, and the force is increased, eventually the box can either slip or tip. Ideally, the box's three different dimensions would be quite different, like a cereal box. Also the friction between the box and the surface it rests on can be altered, for example by placing different fabrics under the box. Students will investigate the influence of a variety of conditions on the outcome of this simple experiment and derive a general method for predicting the box's fate. Students will determine: (i) all of the forces on a box subjected to a horizontally applied force and (ii) whether a horizontally applied force causes a body to slip or tip.

LEARNING OBJECTIVE: Students will use their knowledge of forces and moments and friction to predict whether a box will slip or tip when pushed.

(i) all of the forces on a box subjected to a horizontally applied force: Place a box in an upright position, so that it would tip if the applied force was large enough. Imagine a horizontal force applied to the box that is not large enough to cause it to tip or slip.



Draw an FBD of the box, taking care to label all distances and forces: F_{push} , Weight (W), Normal (F_N) and Friction (F_f). In particular, use an "x" to denote the horizontal distance of the normal force from the center of the box.

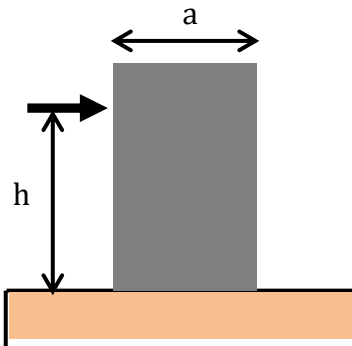
Treating the weight and pushing force symbolically, use the sums of forces and moments to determine mathematical expressions for the normal force, friction force, and the location of the normal force.

For the case in which the box does not slip determine an inequality that relates F_{push} , weight W , and the friction coefficient. (Dimensions may or may not matter.)

For the case in which the box does not tip determine an inequality that relates F_{push} , and the weight W . (Dimensions may or may not matter.)

(ii) whether a horizontally applied force causes a body to slip or tip: Place a tall box with three quite different dimensions on a flat surface. Obtain three fabrics which differ in terms of the amount of friction each can provide to a box sliding across the fabric's surface. Place one of the fabric samples between the box and the horizontal surface (*i.e.* under the box). Apply an increasing horizontal force until the box either slips or tips. Investigate how each of the following conditions influences whether the box first slips or tips.

- Friction at surface, for example, with different fabrics
- Orientation of the box (possibly three different orientations)
- Position of force application relative the bottom



Use the results from the force and moment summations in part (i) to explain the influence of friction, box orientation, and position of force on the tendency to slip or tip first.